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Two-loop light fermion contribution to Higgs production and decays

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Abstract

We compute the electroweak corrections due to the light fermions to the production cross section $\sigma(gg \rightarrow H)$ and to the partial decay widths $\Gamma(H \rightarrow \gamma\gamma)$ and $\Gamma(H \rightarrow gg)$. We present analytic results for these corrections that are expressed in terms of Generalized Harmonic Polylogarithms. We find that for the gluon fusion production cross section and for the decay width $\Gamma(H \rightarrow gg)$ the corrections are large in the Higgs mass region below 160 GeV where they reach up to 9% of the lowest order term. For the decay width $\Gamma(H \rightarrow \gamma\gamma)$ the corrections for Higgs mass above 160 GeV can reach -10% of the lowest order term.

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One of the great successes of the experimental program carried out at LEP has been to put a firm lower bound on the Higgs mass, $m_H > 114$ GeV [1], and at the same time, together with the information coming from SLD, to give a strong indirect evidence that the Higgs boson, the still missing particle of the Standard Model (SM), should be relatively light with a high probability for its mass to be below 200 GeV. The search for the Higgs boson is one of the main objective of the Tevatron and the future Large Hadron Collider (LHC), that are supposed to span all the Higgs mass regions up to 1 TeV. At hadron colliders the main Higgs production mechanism is the gluon fusion [2], a process whose knowledge is fundamental in order to put limits on the Higgs mass or, in case the Higgs is discovered, to compare the measured cross section with the SM result. Concerning the Higgs decay channels, it is quite difficult for an hadron collider to access part of the mass range favored by the LEP results, the so called intermediate Higgs mass region $114 \lesssim m_H \lesssim 160$ GeV, because of the large QCD background to the dominant modes. In this region the rare decay $H \rightarrow \gamma\gamma$ is the most interesting alternative to the usual decay channels.

It is then natural to try to have a theoretical prediction of the gluon fusion production cross section as well as this decay mode, as precise as possible. In the recent years most of the attention to these processes has been devoted to the calculation of the QCD corrections, but we have now reached a stage such that also the electroweak effects can be interesting. In this report we make a first step in the calculation of the electroweak corrections by presenting the analytic result for the two-loop contributions induced by the light (assumed massless) fermions. This subset of corrections, finite and gauge invariant, is actually interesting for two reasons: i) it could be numerically not irrelevant because one has to sum over the generations; ii) because of the presence in the diagrams of two mass scales, m_H and m_W , that in the most physically interesting Higgs mass range are relatively close, and of cuts connected to the zero mass fermions, this subset cannot be computed with the standard heavy mass or momentum expansion method but requires a complete calculation, making it a good probe of new techniques for the computations of two-loop integrals.

We start analyzing the production cross section $\sigma(gg \rightarrow H)$. Because of its importance this process has been investigated with great accuracy in the recent years and it is now known at the next-to-next-to-leading order in QCD [3, 4, 5]. A recent discussion [6] on the residual theoretical uncertainty from perturbative QCD contributions estimate it to be below 10% for $m_H < 200$ GeV. Electroweak corrections to this production mechanism were only considered in the large- m_t limit and found to give a very small effect, below 1% [7].

The Higgs boson, carrying no color, has no tree-level coupling to gluons; therefore this process proceeds via loops. At the partonic level the cross section, not corrected by QCD effects, can be written as:

$$\sigma(gg \rightarrow H) = \frac{G_\mu \alpha_s^2}{512 \sqrt{2} \pi} |\mathcal{G}|^2, \quad (1)$$

where the lowest order one-loop contribution is only due to the top quark and is given by:

$$\mathcal{G}_t^{\text{1l}} = -4 t_H \left[2 - (1 - 4 t_H) H \left(-r, -r; -\frac{1}{t_H} \right) \right], \quad (2)$$

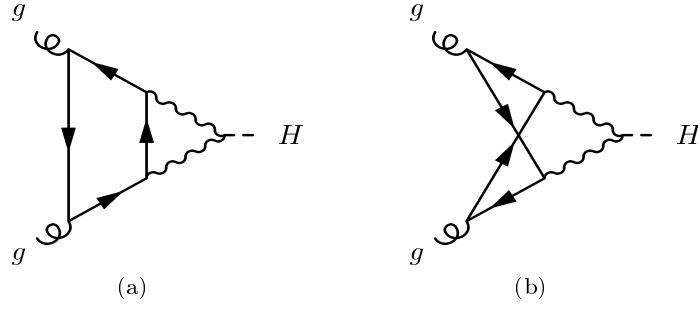


Figure 1: Two-loop topologies involving light fermions contributing to $gg \rightarrow H$.

with $t_H \equiv m_t^2/m_H^2$ and¹

$$H(-r, -r; x) = \frac{1}{2} \log^2 \left(\frac{\sqrt{x+4} - \sqrt{x}}{\sqrt{x+4} + \sqrt{x}} \right). \quad (3)$$

At one-loop the contribution of light fermions is suppressed by their coupling to the Higgs and it is completely negligible.

At the two-loop level the light fermions can contribute to this process because their Higgs coupling suppression can be avoided by coupling them to the W or Z bosons that can directly couple to the Higgs particle. Indeed the light fermions contribute to the Higgs production through the topologies (a) and (b) depicted in Fig. 1, with the W or Z boson exchanged in the loops together with quarks.

The general structure of the amplitude for the production of a Higgs particle, in the fusion process of two gluons of polarization vectors $\epsilon_\mu(q_1)$ and $\epsilon_\nu(q_2)$, can be written as:

$$T^{\mu\nu} = q_1^\mu q_1^\nu T_1 + q_2^\mu q_2^\nu T_2 + q_1^\mu q_2^\nu T_3 + q_1^\nu q_2^\mu T_4 + (q_1 \cdot q_2) g^{\mu\nu} T_5 + \epsilon^{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma} T_6, \quad (4)$$

with gauge invariance requiring that $T_1 = T_2 = 0$ and $T_4 = -T_5$ while T_3 does not contribute for on-shell gluons and T_6 vanishes exactly. The relevant form factors T_4 and T_5 are extracted from the one-particle irreducible (1PI) diagrams of Fig. 1 with the use of standard projectors. The evaluation of the 1PI part of the form factors is performed in few steps. First, the scalar amplitudes are reduced to a set of linearly independent ones. This step is achieved in two different ways [8]: i) a reduction in terms of scalar integrals containing only denominators is obtained with the introduction of a fictitious auxiliary planar double box diagram [9], with final momenta equal to the initial ones, that contains an “extra” propagator with respect to the diagrams of Fig. 1, such that the seven possible invariants that can appear in the numerator of a scalar amplitude can be simplified against the seven propagators of the auxiliary diagram. ii) By shifting the loop momenta in an appropriate way we write the scalar amplitudes in terms of integrals with a number of propagators equal or smaller than the original one, having a set of independent scalar products in the numerators [10]. Next, the independent scalar amplitudes obtained in the two reduction methods are expressed through the use of integration-by-parts identities [11] as a linear combination of the same finite set of basic integrals, called master integrals (MIs). The subsequent evaluation of the MIs is performed employing the technique of the

¹All the analytic continuations are obtained with the replacement $x \rightarrow x - i\epsilon$

differential equations in the external kinematical variables [12]. Finally, the coefficients of the MIs Laurent expansion in $\epsilon = (4-n)/2$, where n is the dimension of the space-time, are expressed in closed analytic form. The 1PI diagrams of Fig. 1 show thresholds in the external Higgs momentum, q^2 , at $q^2 = 0, m_W^2, 4m_W^2$. Because of the presence of the $q^2 = m_W^2$ and $q^2 = 4m_W^2$ thresholds our analytic result cannot be expressed only in terms of Harmonic Polylogarithms (HPLs) [13, 14], a generalization of Nielsen's polylogarithms, but requires a recently introduced extension of the HPLs, the so-called Generalized Harmonic Polylogarithms (GHPLs) [15]. We explicitly checked that our analytic result satisfies all gauge-invariance requests.

The total light fermion contribution to the partonic production cross section $\sigma(gg \rightarrow H)$ is obtained summing over the first two generations of quarks both the diagrams with the W and those with Z bosons. For the bottom quark instead, we can include only the contribution due to the Z boson, which is by itself finite and gauge invariant. We find, in the same normalization of Eq. (1) and in units $\alpha/(2\pi s^2)(m_W^2/m_H^2)$:

$$\mathcal{G}_{lf}^{2l} = \frac{2}{c^4} \left(\frac{5}{4} - \frac{7}{3} s^2 + \frac{22}{9} s^4 \right) A_1[z_H] + 4 A_1[w_H] , \quad (5)$$

where $w_H \equiv m_W^2/m_H^2$, $z_H \equiv m_Z^2/m_H^2$, $s^2 \equiv \sin^2 \theta_W$, $c^2 = 1 - s^2$ and

$$\begin{aligned} A_1[x] = & -4 + 2(1-x) H\left(-1; -\frac{1}{x}\right) - 2x H\left(0, -1; -\frac{1}{x}\right) + 2(1-3x) H\left(0, 0, -1; -\frac{1}{x}\right) \\ & + 2(1-2x) H\left(0, -r, -r; -\frac{1}{x}\right) - 3(1-2x) H\left(-r, -r, -1; -\frac{1}{x}\right) \\ & - \sqrt{1-4x} \left[2 H\left(-r; -\frac{1}{x}\right) - 3(1-2x) H\left(-4, -r, -1; -\frac{1}{x}\right) \right. \\ & \left. + 2(1-2x) H\left(-r, 0, -1; -\frac{1}{x}\right) + 2(1-2x) H\left(-r, -r, -r; -\frac{1}{x}\right) \right] . \end{aligned} \quad (6)$$

Eq. (6) is expressed in terms of ordinary HPLs, the H functions with only indices 0, -1 , and of GHPLs, H functions where also the indices $-r$ and -4 are present. Their explicit expressions are presented in Appendix A.

In Fig. 2 we plot the relative corrections δ , induced by the two-loop light fermion contribution, to the Higgs production cross section $\sigma \equiv \sigma_0(1 + \delta)$, where σ_0 is the lowest order result. The correction raises till the opening of the $2W$ threshold in the topologies (a, b) of Fig. 1, reaching up to 9% of the one-loop result. As soon as the threshold is passed the correction has a sharp decrease with only a small bump at the opening of the $2Z$ threshold, and for values of m_H larger than 200 GeV remains always smaller than 2%. The large (for an electroweak correction) result found in the intermediate Higgs mass region can be explained noticing that the topologies (a, b) of Fig. 1 with their thresholds at $2W$ and $2Z$ are of a new type with respect to the one-loop result that is described by a top loop triangle diagram. We notice that, in the region below the $2W$ threshold, the size of the electroweak light fermion corrections is comparable to the accuracy reached in the knowledge of the QCD corrections to this production cross section [6].

The techniques used to compute the two-loop light fermion corrections to the gluon fusion amplitude can be applied to study also the decay processes $H \rightarrow \gamma\gamma, gg$. The

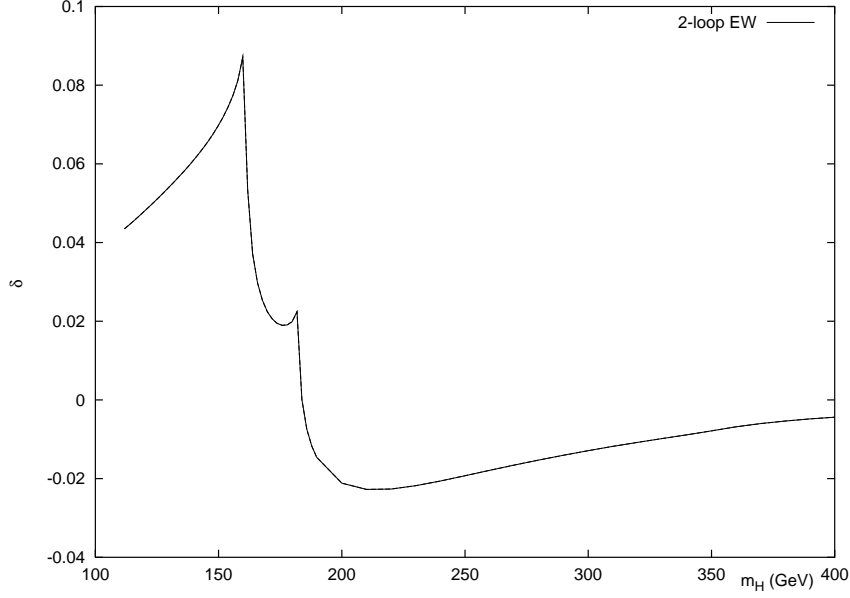


Figure 2: Relative corrections to the production cross section $\sigma(gg \rightarrow H)$ and to the decay width $\Gamma(H \rightarrow gg)$.

result for the latter is very easy to derive because one can relate the partial decay width $\Gamma(H \rightarrow gg)$ to $\sigma(gg \rightarrow H)$ via:

$$\Gamma(H \rightarrow gg) = \frac{8m_H^3}{\pi^2} \sigma(gg \rightarrow H). \quad (7)$$

Thus our result for the light fermion corrections to the production cross section can be translated into a result for the corrections to the partial decay width $\Gamma(H \rightarrow gg)$ and Fig. 2 can also be read as the relative corrections to this decay mode. It should also be noticed that the QCD corrections to this decay are extremely large [16, 4]. As an example, in the intermediate Higgs mass region they shift the decay width by about 60-70% upward.

We consider now the decay $H \rightarrow \gamma\gamma$ that is of particular interest for the Higgs searches at hadron colliders thanks to its clean signature. The Higgs boson, being a neutral particle, has no tree-level coupling to photons, therefore also this decay can proceed only via loops. The corresponding partial decay width can be written as:

$$\Gamma(H \rightarrow \gamma\gamma) = \frac{G_\mu \alpha^2 m_H^3}{128 \sqrt{2} \pi^3} |\mathcal{F}|^2, \quad (8)$$

where the lowest order one-loop contribution is due to W boson and top loops, and is given by [17]:

$$\mathcal{F}^{1l} = \mathcal{F}_W^{1l} + \frac{4}{9} N_c \mathcal{F}_t^{1l}, \quad (9)$$

$$\mathcal{F}_W^{1l} = 2(1 + 6w_H) - 12w_H(1 - 2w_H) H \left(-r, -r; -\frac{1}{w_H} \right), \quad (10)$$

$$\mathcal{F}_t^{1l} = \mathcal{G}^{1l}, \quad (11)$$

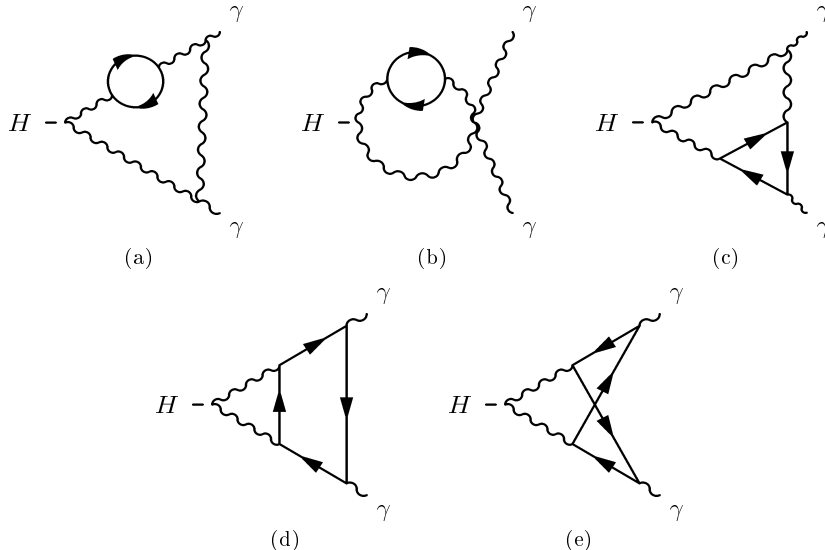


Figure 3: Two-loop topologies involving light fermions contributing to $H \rightarrow \gamma\gamma$.

with N_c the color factor.

The W and top one-loop contributions interfere destructively. They approach constant values ($\mathcal{F}_W^{1l} \rightarrow 7$, $\mathcal{F}_t^{1l} \rightarrow -4/3$) for mass of the particle inside the loop much heavier than m_H , with the W loops always providing the dominant part for Higgs mass up to $m_H \sim 600$ GeV, where the top contribution becomes comparable.

Two-loop investigations of this decay channel have been mainly devoted to QCD corrections to the one-loop top contribution [18]. Concerning two-loop electroweak corrections only the $\mathcal{O}(G_\mu m_t^2)$ term [19] and the $\mathcal{O}(G_\mu m_H^2)$ one [20] were studied.

Differently from the one-loop level at two-loop the light fermions can contribute to this partial decay width through all diagrams whose topologies are depicted in Fig. 3. The choice of an appropriate gauge-fixing procedure can somewhat reduce the number of diagrams to be computed. To achieve this we find it convenient to employ the Background Field Method (BFM) quantization framework. The BFM is a technique for quantizing gauge theories [21, 22] that avoids the complete explicit breaking of the gauge symmetry. One of the salient features of this approach is that all fields are splitted in two components: a classical background field \hat{V} and a quantum field V that appears only in the loops. The gauge-fixing procedure is achieved through a non linear term in the fields that breaks the gauge invariance only of the quantum part of the lagrangian, preserving the gauge symmetry of the effective action with respect to the background fields. Thus, the BFM gauge-fixing function induces a modification of some of the vertices in which background fields are present, such that, in the BFM Feynman gauge ($\xi_Q = 1$, ξ_Q being the gauge parameter), the vertex coupling a background (external) photon to a quantum W boson and its unphysical scalar counterpart is absent². With this choice of gauge only the physical W and Z bosons are exchanged in Fig. 3. In particular, in the topologies (a, b, c) the internal wavy line can only be a W boson while in the topologies (d, e) it can represent both a W and a Z boson. The evaluation of the diagrams of Fig. 3 follows the line already

²A general discussion on the application of the BFM to the SM with the complete set of BFM Feynman rules can be found in Ref.[23] while a specific two-loop calculation in this framework is presented in Ref.[24].

discussed for the gluon fusion cross section. Concerning the counterterm diagrams, which are not depicted in Fig. 3, their evaluation does not present any particular difficulty. We just notice that the contributions coming from the $\mathcal{O}(\epsilon)$ part of the one-loop diagrams cancel out in the total result. The renormalization is actually straightforward. Because we are in the presence of two real photons their coupling is given by $\alpha = 1/137.036$ while the coupling of the Higgs to the vector boson is more appropriately described in terms of G_μ . Correspondingly, we employ the on-shell definition for the W boson mass.

The full light fermion contribution to the Higgs decay into two photons is obtained summing all the diagrams over the 3 lepton families plus the two first generations of quarks. Concerning the third generation, the bottom contribution to topologies (d, e) mediated by the Z boson, that is finite and gauge-invariant, can also be included. The light fermion contribution to the partial width $H \rightarrow \gamma \gamma$, in the same normalization of Eq. (8), reads in units $\alpha/(2\pi s^2)(m_W^2/m_H^2)$:

$$\begin{aligned} \mathcal{F}_{lf}^{2l} = & 2 N_c A_2 [-2/9, w_H] + 3 A_2 [0, w_H] + \frac{2 N_c}{c^4} \left(\frac{11}{36} - \frac{19}{27} s^2 + \frac{70}{81} s^4 \right) A_1 [z_H] \\ & + \frac{3}{c^4} \left(\frac{1}{2} - 2 s^2 + 4 s^4 \right) A_1 [z_H] , \end{aligned} \quad (12)$$

where

$$\begin{aligned} A_2[q, x] = & -8(1+q) + 4(1+q)(1-x) H\left(-1; -\frac{1}{x}\right) - 2(1+2qx) H\left(0, -1; -\frac{1}{x}\right) \\ & - \frac{2}{3}(5-12x) H\left(-r, -r; -\frac{1}{x}\right) - 6(1+q-3x-2qx) H\left(-r, -r, -1; -\frac{1}{x}\right) \\ & + 2(1+2q) \left[(1-2x) H\left(0, -r, -r; -\frac{1}{x}\right) + (1-3x) H\left(0, 0, -1; -\frac{1}{x}\right) \right] \\ & - \sqrt{1-4x} \left\{ 2(1+2q) H\left(-r; -\frac{1}{x}\right) - 6q(1-2x) H\left(-4, -r, -1; -\frac{1}{x}\right) \right. \\ & \quad \left. + 4q(1-2x) \left[H\left(-r, 0, -1; -\frac{1}{x}\right) + H\left(-r, -r, -r; -\frac{1}{x}\right) \right] \right\} \\ & + \frac{6(1-2x)^2}{\sqrt{1-4x}} H\left(-r, -1; -\frac{1}{x}\right) , \end{aligned} \quad (13)$$

and A_1 has been given in Eq. (6).

Eq. (13) shows an unphysical singularity at $x = \frac{1}{4}$, i.e. at the opening of the $2W$ threshold, connected to the appearance in its last line of a term with a square root in the denominator that exactly at the threshold becomes formally infinite. This unphysical infinity is actually a signal that our first order treatment of the W propagator in topologies (a, b) of Fig. 3 is inadequate in the $2W$ threshold region. To obtain a finite result also in this region we regulate the singularity by performing the replacement $m_W \rightarrow m_W - i\Gamma_W/2$ in the square root and we check the dependence of the final result on the regulator.

In Fig. 4 we plot separately the relative corrections, δ , to the decay width $\Gamma \equiv \Gamma_0(1 + \delta)$ ($\Gamma_0 \propto |\mathcal{F}^{1l}|^2$) induced by the two-loop light fermion contribution and by the QCD corrections to the one-loop top term as well as the sum of the two contributions. The

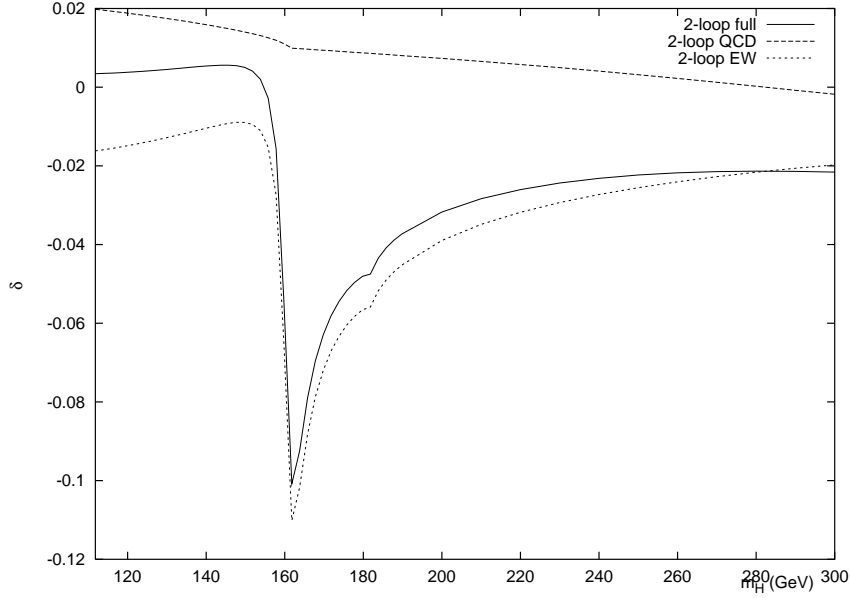


Figure 4: Relative corrections to the decay width $H \rightarrow \gamma\gamma$.

figure is obtained by regulating the singularity at $x = \frac{1}{4}$ with $\Gamma_w/m_w = 2.5 \cdot 10^{-2}$. We check the dependence of the result on the regulator by varying Γ_w/m_w down to $2.5 \cdot 10^{-9}$. Above the threshold already at $m_H = 165$ GeV the result changes by less than 5% and becomes insensitive to the size of the regulator at $m_H = 170$ GeV. Below the threshold, down to $m_H = 150$ GeV the variation is larger. However, for a Higgs mass value around 150 GeV the light fermion correction is small and therefore it is not surprising that the relative variations are large. It is clear that in the region $150 \lesssim m_H \lesssim 170$ GeV our result should be taken with some caution.

As it can be seen from Fig. 4 the QCD and light fermion contributions for $m_H \lesssim 140$ GeV are of comparable size but with opposite sign so that the total correction is below 1%. As an example, for $m_H = 120$ GeV the light fermion contribution amounts to $\sim(-1.5\%)$ of the total width while the QCD one gives $\sim(+1.8\%)$. For values of m_H above 140 GeV the light fermion correction becomes small until the region close to the opening of the $2W$ threshold is reached. Crossing the threshold the correction becomes quite large, in absolute value, but as soon as the $2W$ threshold region is passed it starts to decrease again. However, for m_H around 200 GeV, we find still a $\sim 5\%$ correction while the QCD contribution is relatively small. The visible bump in the figure around 180 GeV is actually connected with the opening of the $2Z$ threshold in the topologies (d, e) of Fig. 3.

In conclusion, we have computed the two-loop electroweak corrections induced by the light fermions to the Higgs gluon fusion production cross section $\sigma(gg \rightarrow H)$ and to the partial decay widths $\Gamma(H \rightarrow \gamma\gamma)$ and $\Gamma(H \rightarrow gg)$. For the gluon fusion production cross section we have found that the light fermion corrections are quite large in the region below the $2W$ threshold reaching up to 9% of the lowest order result. In the case of the Higgs decay to two photons, the corrections are large above the $2W$ threshold, while below it they are quite small a part the region $m_H \lesssim 140$ GeV where they are larger

than 1% cancelling part of the QCD corrections giving a total correction below 1%. It should be recalled that, both for the production and the decay, the light fermions are only a small part of the electroweak corrections. A complete calculation of the electroweak corrections to $\sigma(gg \rightarrow H)$ seems to be relevant to assure the accuracy of the knowledge of this important production cross section.

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A Generalized Harmonic Polylogarithms

In this Appendix, we outline the definitions and properties of the HPLs and GHPLs and give the explicit expressions of those entering in Eqs. (6,13).

The idea lying behind the introduction of HPLs is to express a given integral coming from the calculation of a Feynman diagram in a unique and non-redundant way as a linear combination of a minimal set of independent transcendental functions. These functions are expressed as repeated integrations over a starting set of basis functions and this set depends strongly on the problem one has to solve, being connected directly to the threshold structure of the diagrams under consideration. In the case in which only thresholds at 0 and $4m^2$ or at 0 and m^2 occur, the original set of HPLs introduced in [13] is sufficient to cover all the possible integrals. However, for different structure of the thresholds of the Feynman diagrams involved in the calculation, as it is the case at hand, one is forced to enlarge the set of starting basis functions.

In our case, we enlarge the original basis functions with indices $\{-1, 0, 1\}$:

$$g(-1; x) = \frac{1}{1+x}, \quad g(0; x) = \frac{1}{x}, \quad g(1; x) = \frac{1}{1-x}; \quad (14)$$

that give a “weight-1” HPLs:

$$H(-1; x) = -\log(1-x), \quad H(0; x) = \log x, \quad H(1; x) = \log(1+x), \quad (15)$$

with the following two basis functions [15]:

$$g(-4; x) = \frac{1}{4+x}, \quad g(-r; x) = \frac{1}{\sqrt{x(4+x)}}, \quad (16)$$

with the corresponding weight-1 GHPLs:

$$H(-4; x) = \log(4+x) - 2\log 2, \quad H(-r; x) = -\log z, \quad (17)$$

where:

$$z = \frac{\sqrt{x+4} - \sqrt{x}}{\sqrt{x+4} + \sqrt{x}}. \quad (18)$$

Integrals with weight > 1 are defined using the recursive definitions:

$$H(\vec{0}_w; x) = \frac{1}{w} \log^w x, \quad H(a, \vec{w}; x) = \int_0^x dt g(a; t) H(\vec{w}; t), \quad (19)$$

where $\vec{0}_w = (0, 0, \dots, 0)$ is a vector containing w zeroes, \vec{w} is a vector with w components running over the set $\{-4, -r, -1, 0, 1\}$ and a takes the values $-4, -r, -1, 0, 1$.

It has been shown [13, 14, 15] that the set of (G)HPLs constructed in this way satisfies useful relation among themselves. As an example, the product of two GHPLs of weight w_1 and w_2 can be expressed as a linear combination of GHPLs of weight $w_1 + w_2$.

Eqs. (6,13) contain 11 GHPLs. $H(-r, -r; x)$ is defined in Eq. (3) while $H(-1; x)$ and $H(-r; x)$ have already been presented in Eqs. (15,17); the other 8 are:

$$H(0, -1; x) = -\text{Li}_2(-x), \quad (20)$$

$$H(0, 0, -1; x) = -\text{Li}_3(-x), \quad (21)$$

$$H(-r, -r, -r; x) = -\frac{1}{6} \log^3 z, \quad (22)$$

$$H(0, -r, -r; x) = -\frac{1}{6} \log^3 z + 2 \text{S}_{1,2}(1 - z) \quad (23)$$

$$H(-r, -1; x) = \int_0^x \frac{dt}{\sqrt{t(4+t)}} \log(1+t), \quad (24)$$

$$H(-r, 0, -1; x) = \int_0^x \frac{dt}{\sqrt{t(4+t)}} \int_0^t \frac{dq}{q} \log(1+q), \quad (25)$$

$$H(-r, -r, -1; x) = \int_0^x \frac{dt}{\sqrt{t(4+t)}} \int_0^t \frac{dq}{\sqrt{q(4+q)}} \log(1+q), \quad (26)$$

$$H(-4, -r, -1; x) = \int_0^x \frac{dt}{(4+t)} \int_0^t \frac{dq}{\sqrt{q(4+q)}} \log(1+q). \quad (27)$$

where $\text{Li}_2(x) = -\int_0^x dt \frac{\log(1-t)}{t}$, and $\text{Li}_3(x) = \int_0^x dt \frac{\text{Li}_2(t)}{t}$ are the Nielsen's polylogarithms and $\text{S}_{1,2}(1-z) = \frac{1}{2} \int_0^{1-z} dt \frac{\log^2(1-t)}{t}$ is a Spence function.

These 11 GHPLs are real and analytic in all the half plane $x \geq 0$ and they present cuts in the plain $x < 0$. In our case the variable x is defined as $x = -s/m^2$, where s is the squared c.m. energy and m is the mass of the particle running in the loop (W or Z bosons). In the physical region $s = m_H^2 > 0$, the variable x is negative, and the GHPLs have to be continued analytically. The analytical continuation is done with the usual $i\epsilon$ -prescription, i.e. giving a small positive imaginary part to s : $s + i\epsilon$. In so doing, we have:

$$x \rightarrow -y - i\epsilon, \quad (28)$$

where $y = s/m^2 > 0$, and, for example:

$$\sqrt{x} \rightarrow \sqrt{-y - i\epsilon} = -i\sqrt{y}, \quad (29)$$

$$\log x \rightarrow \log(-y - i\epsilon) = \log y - i\pi. \quad (30)$$

The numerical evaluation of the 11 GHPLs comes straightforwardly from their definition as repeated integrals.

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